**Hong Kong Mathematics Olympiad (2009 / 2010)**

**Final Event 1 (Group)**

**香港数学竞赛 (2009 / 2010)**

**决赛项目 1 (团体)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求 $\sin^2 1° + \sin^2 2° + \cdots + \sin^2 89°$ 的值。

   Find the value of $\sin^2 1° + \sin^2 2° + \cdots + \sin^2 89°$.

2. 已知 $\frac{x+z}{2z-x} = \frac{z+2y}{2x-z} = \frac{x}{y}$。求 $\frac{x}{y}$ 的值。

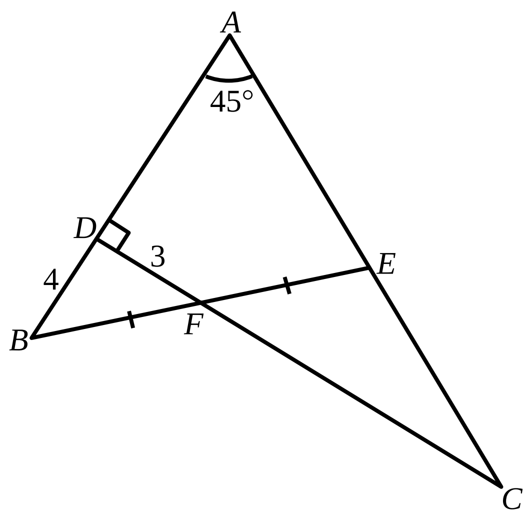
   Given that $\frac{x+z}{2z-x} = \frac{z+2y}{2x-z} = \frac{x}{y}$. Find the value of $\frac{x}{y}$.

3. 求方程 $(2^x-4)^3 + (4^x-2)^3 = (4^x+2^x-6)^3$ 的所有实根 $x$ 的总和。

   Find the sum of all real roots $x$ of the equation
   $(2^x-4)^3 + (4^x-2)^3 = (4^x+2^x-6)^3$.

4. 在图一，若 $AB \perp CD$， $F$ 是 $BE$ 的中点，$\angle A = 45°$，$DF = 3$，$BD = 4$ 及 $AD = n$，求 $n$ 的值。

   In Figure 1, if $AB \perp CD$, $F$ is the midpoint of $BE$, $\angle A = 45°$, $DF = 3$, $BD = 4$ and $AD = n$, find the value of $n$.




**图一**

**Figure 1**

**Hong Kong Mathematics Olympiad (2009 / 2010)**

**Final Event 2 (Group)**

**香港数学竞赛 (2009 / 2010)**

**决赛项目 2 (团体)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 $p=2-2^2-2^3-2^4-\ldots-2^9-2^{10}+2^{11}$，求 $p$ 的值。

   If $p=2-2^2-2^3-2^4-\ldots-2^9-2^{10}+2^{11}$, find the value of $p$.

2. 已知 $x$ , $y$ , $z$ 为 3 个相异实数。若 $x+\frac{1}{y}=y+\frac{1}{z}=z+\frac{1}{x}$ 及 $m=x^2y^2z^2$。求 $m$ 的值。

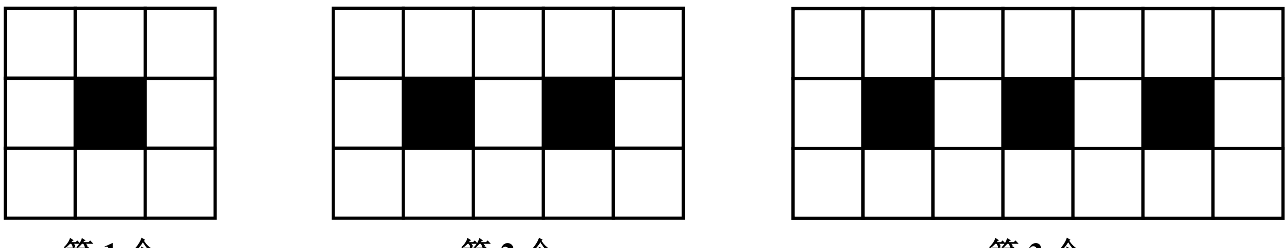
   Given that $x$ , $y$ , $z$ are three distinct real numbers. If $x+\frac{1}{y}=y+\frac{1}{z}=z+\frac{1}{x}$ and $m=x^2y^2z^2$, find the value of $m$.

3. 已知 $x$ 为一正实数，且满足 $x\cdot 3^x=3^{18}$。若 $k$ 是一正整数且 $k<x<k+1$，求 $k$ 的值。

   Given that $x$ is a positive real number and $x\cdot 3^x=3^{18}$. If $k$ is a positive integer and $k<x<k+1$, find the value of $k$.

4. 图一所示为利用黑白两种颜色凑成有规律的图形。求第 95 个图形的白色格子的数目。

   Figure 1 shows the sequence of figures that are made of squares of white and black.
   Find the number of white squares in the 95th figure.



**第 1 个** **1st figure** **第 2 个** **2nd figure** **第 3 个** **3rd figure**

**图一 Figure 1**

**Hong Kong Mathematics Olympiad (2009 / 2010)**

**Final Event 3 (Group)**

**香港数学竞赛 (2009 / 2010)**

**决赛项目 3 (团体)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求 $101^{303}+303^{101}$ 的最小质因子。

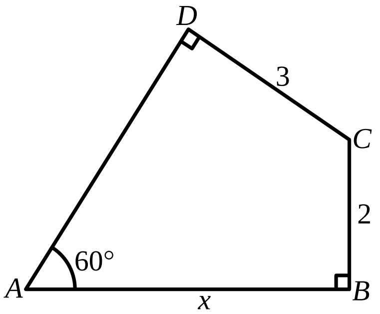
Find the smallest prime factor of $101^{303}+303^{101}$.

2. 设 $n$ 为 $\dfrac{1}{\frac{1}{1980}+\frac{1}{1981}+\cdots+\frac{1}{2009}}$ 的整数部分，求 $n$ 的值。

Let $n$ be the integral part of $\dfrac{1}{\frac{1}{1980}+\frac{1}{1981}+\cdots+\frac{1}{2009}}$, find the value of $n$.

3. 在图一中，若 $\angle A=60°$、$\angle B=\angle D=90°$、 $BC=2$、 $CD=3$ 及 $AB=x$，求 $x$ 的值。

In Figure 1, if $\angle A=60°$, $\angle B=\angle D=90°$, $BC=2$, $CD=3$ and $AB=x$, find the value of $x$.




**图一 Figure 1**

4. 已知函数 $f$ 对所有实数 $x$ 皆满足 $f(2+x)=f(2-x)$，且 $f(x)=0$ 恰好有四个相异实根。求这四个相异实根之和。

Given that the function $f$ satisfies $f(2+x)=f(2-x)$ for every real number $x$ and that $f(x)=0$ has exactly four distinct real roots. Find the sum of these four distinct real roots.

# Hong Kong Mathematics Olympiad (2009 / 2010)
## Final Event 4 (Group)
# 香港数学竞赛 (2009 / 2010)
## 决赛项目 4 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知方程 $(a-1)x^2-mx+a=0$ 的两根均为正整数。求 $m$ 的值。

Given that the equation $(a-1)x^2-mx+a=0$ has two roots which are positive integers. Find the value of $m$.

2. 已知 $x$ 为一实数及 $y=\sqrt{x^2-2x+2}+\sqrt{x^2-10x+34}$。求 $y$ 的最小值。

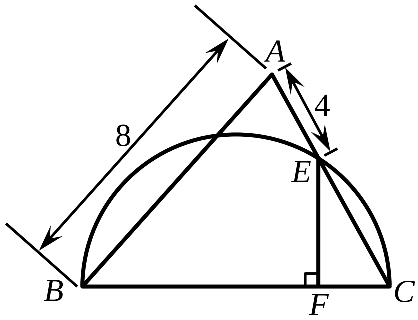
Given that $x$ is a real number and $y=\sqrt{x^2-2x+2}+\sqrt{x^2-10x+34}$. Find the minimum value of $y$.

3. 已知 $A$、$B$、$C$ 为正整数，且 $A$、$B$ 和 $C$ 的最大公因子等于 1。
若 $A$、$B$、$C$ 满足 $A\log_{500}5+B\log_{500}2=C$，求 $A+B+C$ 的值。

Given that $A$, $B$, $C$ are positive integers with their greatest common divisor equal to 1.
If $A$, $B$, $C$ satisfy $A\log_{500}5+B\log_{500}2=C$, find the value of $A+B+C$.

4. 在图一中，$BEC$ 是一半圆形及 $F$ 是直徑 $BC$ 上的一点。已知 $BF:FC=3:1$，$AB=8$ 及 $AE=4$。求 $EC$ 的长度。

In Figure 1, $BEC$ is a semicircle and $F$ is a point on the diameter $BC$. Given that $BF:FC=3:1$, $AB=8$ and $AE=4$. Find the length of $EC$.



**图一 Figure 1**